# Financial Algebra
# Summer Assignment

**The following packet contains topics and definitions that you will be required to know in order to succeed in Financial Algebra this year. You are advised to be familiar with each of the concepts and to complete the included problems by the beginning of the school year. All of these topics were discussed in Geometry or Algebra II and will be used frequently throughout the year.**

## Review

Things to Remember:

Rational numbers are any real numbers that can be expressed as the ratio of two integers. This includes all terminating and all repeating decimals.

Examples:

$\frac{2}{5} = 0.4$ $\qquad \frac{7}{1} = 7$ $\qquad \frac{7}{8} = 0.875$ $\qquad \frac{1}{3} = 0.333\ldots$

Irrational numbers are any real numbers that cannot be expressed as the ratio of two integers. This will include all non-terminating, non-repeating decimals.

Examples:

$\sqrt{2} \approx 1.4142135623\ldots$ $\qquad \pi \approx 3.1415926535\ldots$ $\qquad e \approx 2.7182818284$

Properties of Inequalities:

Let a, b, c and d be real numbers.

1. Transitive property: a<b and b<c → a < c
2. Adding inequalities: a<b and c<d → a+c < b+c, c > 0
3. Multiplying by a positive constant: a < b → ac < bc, c > 0
4. Multiplying by a negative constant: a<b → ac > bc, c < 0
5. Adding a constant: a < b → a + c < b + c
6. Subtracting a constant: a < b → a – c < b – c

Definition of Absolute Value:

The absolute value of a real number a is:

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

Properties of Absolute Value:

1. Multiplication: $|ab| = |a||b|$
2. Division: $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$
3. Power: $|a^n| = |a|^n$
4. Square Root: $\sqrt{a^2} = |a|$

Distance Between Two Points:

The distance d between any two points $x_1$ and $x_2$ on a real number line is:

$$d = |x_1 - x_2| = \sqrt{(x_2 - x_1)^2}$$

The distance d between any two points $(x_1, y_1)$ and $(x_2, y_2)$ on a Cartesian plane is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Find the distance between each pair of points:**

**1) (6, -2) and (-5, 12)**

**2) (5.8, -1) and (7, -4)**

Midpoints:

The midpoint of the interval with endpoints a and b is found by taking the average of the endpoints.

$$M = \frac{a+b}{2}$$

The midpoint of a segment with endpoints at $(x_1, y_1)$ and $(x_2, y_2)$ is found by taking the average of the two coordinate values.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Find the midpoint of each segment with the indicated endpoints:**

**3) (-4, 3) and (7, 12)**

**4) (10, -3) and (-14, -9)**

Properties of Exponents:

1. Whole number exponents: $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ (n factors of x)
2. Zero exponents: $x^0 = 1, x \neq 0$
3. Negative Exponents: $x^{-n} = \frac{1}{x^n}$
4. Radicals (principal nth root): $\sqrt[n]{x} = a \rightarrow x = a^n$
5. Rational exponents: $x^{1/n} = \sqrt[n]{x}$
6. Rational exponents: $x^{m/n} = \sqrt[n]{x^m}$

Operations with Exponents:

1. Multiplying like bases: $x^n x^m = x^{m+n}$
2. Dividing like bases: $\frac{x^m}{x^n} = x^{m-n}$
3. Removing parentheses: $(xy)^n = x^n y^n$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $(x^n)^m = x^{nm}$

**Simplify each of the following expressions:**

**5)** $\boldsymbol{2a^2b^{-4} \cdot 4a^{-8}b^6}$

**6)** $\boldsymbol{\frac{8a^2b^{-2}}{4a^4c^{-5}}}$

**7)** $\boldsymbol{\left(\frac{3x^4}{y^{-2}}\right)^3}$

Special Products and Factorization Techniques

Quadratic Formula:

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Solve the following equations:**

**8)** $\boldsymbol{3x^2 - 4x - 2 = 0}$

**9)** $\boldsymbol{-5x^2 + 6x + 4 = 0}$

Special Products:

$$x^2 - a^2 = (x-a)(x+a)$$

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

**Factor each of the following:**

**10)** $\mathbf{4x^2 - 144}$

**11)** $\mathbf{8x^3 - 27}$

**12)** $\mathbf{64x^3 + 125y^3}$

**13)** $\mathbf{x^2 - 13x + 42}$

**14)** $\mathbf{8x^2 - 10x - 3}$

**15)** $\mathbf{4x^3 + 8x^2 - 5x - 10}$

Binomial Theorem:

$$(x+a)^2 = x^2 + 2ax + a^2$$

$$(x-a)^2 = x^2 - 2ax + a^2$$

$$(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$$

**Expand each of the following:**

**16)** $\mathbf{(x+4)^2}$

**17)** $\mathbf{(x-6)^2}$

**18)** $\mathbf{(x+3)^3}$

**19)** $\mathbf{(x-2)^3}$

Lines

| | |
|---|---|
| Slope: | $\frac{y_2-y_1}{x_2-x_1}$ |
| Slope Intercept Form: | y = mx + b |
| Standard Form: | ax + by = c |
| Point-Slope Form: | $y - y_1 = m(x - x_1)$ |

**Use the two points to write the equation of the line in all three forms:**

**20) (2, -5) and (7, 12)**

Transformations

| | |
|---|---|
| Vertical Translations: | $y = f(x) \pm k$ |
| Horizontal Translations: | $y = f(x \pm h)$ |
| Y-axis flip: | $y = f(-x)$ |
| X-axis flip: | $y = -f(x)$ |

**Describe each of the following transformations:**

**21)** $\boldsymbol{f(x) = -x^2 + 4}$

**22)** $\boldsymbol{f(x) = |-x| - 4}$

**23)** $\boldsymbol{f(x) = \sqrt{x - 3}}$

Functions

Domain: a set of all possible values for the independent variable

Range: a set of all possible values for the dependent variable

**Find the domain and range of the following functions:**

**24)** $y = \sqrt{x+1}$

**25)** $y = \frac{3x-4}{4x+10}$

**26)** $y = \begin{cases} 1-x, & x < 1 \\ \sqrt{x-1}, & x \geq 1 \end{cases}$

Even and Odd Functions:

$If\ f(-x) = -f(x), then\ the\ function\ is\ odd$

$If\ f(-x) = f(x), then\ the\ function\ is\ even$

**Determine if the function is even or odd:**

**27)** $y = 3x^2$

**28)** $y = 2x^2 + 4x$

**29)** $y = 4x^3$

End Behavior:

-If the degree of f is even and the lead term coefficient is positive, then the left and right

ends of the function both approach positive infinity.

-If the degree of f is even and the lead term coefficient is negative, then the left and right ends of the function both approach negative infinity.

-If the degree of f is odd and the lead term coefficient is positive, then the left end approaches negative infinity and the right end approaches positive infinity.

-If the degree of f is odd and the lead term coefficient is negative, then the left end approaches positive infinity and the right end approaches negative infinity.

**Describe the end behavior for each of the following functions:**

**30)** $f(x) = -3x^2 + 4x - 2$

**31)** $f(x) = 2x^3 - 3x^2 + 6x - 1$

**32)** $f(x) = 5x^5 - 6$

Functions

**Evaluate each of the following for the function $f(x) = x^2 - 4x + 7$:**

**33)** $f(4) =$

**34)** $f(y^3) =$

**35)** $f(x + y) =$

**Evaluate the following compositions of functions:**

**Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$**

**36)** $f(x) \cdot g(x)$

**37)** $f(g(x))$

**38)** $g(f(x))$

Inverse Functions

In order to calculate an inverse of a function algebraically, you must switch all of the x and y variables and solve the new equation for y. The inverse only exists if the resulting equation is a function.

**Find the inverse (if it exists) of each of the following functions:**

**39)** $f(x) = 3x + 2$

**40)** $f(x) = 2x^2 - 4$

**41)** $f(x) = \sqrt{x + 1}$

Logarithms

Natural Logarithmic Function:

$$\ln x = b \text{ if and only if } e^b = x$$

Inverse Properties of Logarithms:

$$\ln e^x = x \qquad\qquad e^{\ln x} = x$$

**Solve each of the following equations:**

**42)** $\mathbf{10 + e^{0.1x} = 14}$

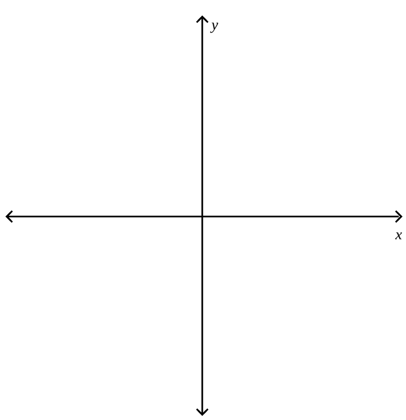
**43)** $\mathbf{3 + 2\ln x = 7}$

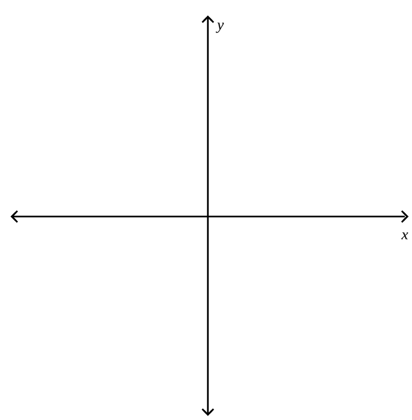
**Graph the following equations without a calculator:**

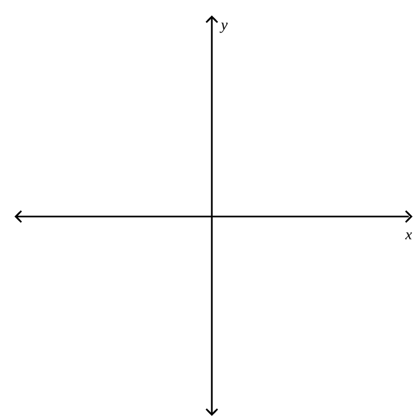
**44)** $\boldsymbol{f(x) = \ln(x-2) + 3}$

**45)** $\boldsymbol{f(x) = -2 + e^{x+1}}$

**46)** $\boldsymbol{f(x) = \begin{cases} \ln x + 1, & x < 1 \\ e^{x-1}, & x \geq 1 \end{cases}}$



Properties of Logarithms

Product Property: $log_b a + log_b c = log_b(ac)$

Quotient Property: $log_b a - log_b c = log_b\left(\frac{a}{c}\right)$

Power Property: $log_b a^c = c \cdot log_b a$

**Condense each of the following to a single logarithm:**

**47)** $\boldsymbol{4lnx + 6lny - lnz}$

**48)** $\boldsymbol{\frac{1}{3}\left[2ln(x+3) + lnx - ln\left(x^2-1\right)\right]}$

**Rewrite the expression as a sum, difference or multiple of logarithms:**

**49)** $\boldsymbol{ln\sqrt{\frac{x^3}{x+1}}}$

**50)** $\boldsymbol{ln\frac{3x(x+1)}{(2x+1)^2}}$

Compound Interest

Let P be the amount deposited, t the number of years, A the balance and r the annual interest rate (in decimal form).

1. Compounded n time per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

2. Compounded continuously: $A = Pe^{rt}$

**Determine the balance A for P dollars invested at rate r for t years, compounded n times per year.**

**51) P=$1000, r=3%, t=10 years**

| t | 1 | 10 | 20 | 30 | continuous |
|---|---|---|---|---|---|
| P | | | | | |

**52) P=$2500, r=5%, t=40 years**

| t | 1 | 10 | 20 | 30 | continuous |
|---|---|---|---|---|---|
| P | | | | | |