

# Review for AP Calculus

## I. Function Notation

Given:  $f(x) = 3x^2 - 7x$        $g(x) = 5x + 4$        $h(x) = \begin{cases} (x+1)^2 & x < 3 \\ 2x+10 & x \geq 3 \end{cases}$

Find the following.

- 1)  $f(-2)$     2)  $6f(4)$     3)  $g(a)$     4)  $f(g(x))$     5)  $f(3-m)$     6)  $h(-7)$   
7)  $h(10)$     8)  $h(3)$     9)  $g(f(x))$     10)  $f(g(2))$     11)  $f(x)g(x)$     12)  $f(x+h)$   
13)  $g(x+h) - g(x)$     14)  $f(-5 + \Delta x) - f(-5)$     15)  $\frac{f(x + \Delta x) - f(x)}{\Delta x}$

## II. Piece-wise Functions

1) Graph  $h(x) = \begin{cases} (x+1)^2 & x < 3 \\ 2x+10 & x \geq 3 \end{cases}$  without a calculator.

2) Graph  $p(x) = \begin{cases} x+5 & x \leq -2 \\ -x+7 & x > 2 \end{cases}$  without a calculator.

3) Find the following:

- a)  $h(-2)$     b)  $h(6)$     c)  $h(3)$     d)  $p(5)$     e)  $p(-2)$     f)  $p(0)$

## III. Lines

- 1) Write the equation of the line that has a slope of 3 and passes through the point  $(7, -4)$ .
- 2) Write the equation of the line that passes through the points  $(2, -6)$  and  $(-3, 5)$ .
- 3) Write the equation of the line that passes through the points  $(-5, 4)$  and  $(7, 4)$ .
- 4) Write the equation of the line that is parallel to  $y = \frac{1}{4}x + 5$  that passes through the point  $(-3, 2)$ .

- 5) Write the equation of the line that is perpendicular to  $y = \frac{7}{3}x + 1$  that passes through the point  $(1, -3)$ .
- 6) Write the equation of the line that is perpendicular to  $y = 7$  that passes through the point  $(8, 7)$ .
- 7) Write an equation of the line through the point
- parallel to the given line
  - perpendicular to the given line

Line:  $3x + 4y = 7$

Point:  $(-6, 4)$

#### IV. Intersection points

- Find the intersection point(s) of  $y = 2x^3 - 7x^2 + x - 10$  and  $y = x^3 - 7x^2 + x - 18$ .
- Find the intersection point(s) of  $y = x^4 - 2x^2 + 1$  and  $y = 1 - x^2$ .

#### V. Factoring.

Factor the following.

- $x^2 - 9$
- $x^2 + 7x + 12$
- $x^2 - x - 42$
- $4x^6 + 8x^3 - 2x^2$
- $6x^2 - 5x - 6$
- $4x^2 - 32x + 60$
- $3x^2 - 300$
- $x^2e^x + 2xe^x$
- $5^x e^x + 5^x e^x \ln 5$
- $6x^2(x-4)^3 + 3x(x-4)^4$
- $48x^2(x^2 + 1) + 12(x^2 + 1)^2$
- $6x(6x+1)^2(x^2 - 5)^2 + 12(x^2 - 5)^3(6x+1)$
- $8\sin^2 x \cos^2 x - 12\sin x \cos^2 x$

## VI. Graphing.

Graph the following "Parent Graphs" without a calculator.

1)  $y = mx + b$       2)  $y = x^2$       3)  $y = x^3$       4)  $y = |x|$       5)  $y = \frac{1}{x}$

6)  $y = \frac{1}{x^2}$       7)  $y = \sqrt{x}$       8)  $y = x^{1/3}$       9)  $y = \sin x$       10)  $y = \cos x$

11)  $y = e^x$       12)  $y = \ln x$

Students should also know how to shift all graphs except # 6,9,and 10

Students should also understand how to state the Domain and Range of a function in interval notation.

## VII. Rational Expressions

Simplify the following.

1)  $\frac{3(x + \Delta x) - 3x}{\Delta x}$       2)  $\frac{(x + h)^2 - 4(x + h) + 5 - (x^2 - 4x + 5)}{h}$       3)  $\frac{3x^2 + 13x - 30}{x + 6}$

4)  $\frac{x^2 - 9x + 18}{x^2 - x - 6}$       5)  $\frac{x + 1}{x^2 - 8x - 9}$       6)  $\frac{3x - 27}{x^2 - x - 72}$       7)  $\frac{7 - x}{x^2 - 49}$

8)  $\frac{1}{\sqrt{x^2 - 4}} \left( \frac{1}{\sqrt{x^2 - 4}} \right) (x - 2)$       9)  $\frac{3x(x^2 + 1) - 3x(x^2 - 1)}{(x^2 + 1)^2}$       10)  $\frac{(x^2 - 1)(-2 - 2x) - 2x(3 - 2x - x^2)}{(x^2 - 1)^2}$

11)  $\frac{1}{\sqrt{x^6 - 10}} \left( \frac{1}{2} \right) (x^6 - 10)^{-1/2} (6x^5)$       12)  $\frac{(3x + 5)(2x - 1) - 4(7x + 1)}{(3x + 5)^2}$       13)  $\frac{1}{x} - \frac{2x}{x^2 + 1}$

14)  $\frac{1}{x^2 + 3} + \frac{3 - x^2}{(x^2 + 3)^2}$       15)  $\frac{-3x^2}{(x^2 + 1)^{3/2}} + \frac{3}{(x^2 + 1)^{1/2}}$       16)  $\frac{1}{x \ln 7} - \frac{1}{(x - 1) \ln 7}$

17)  $\frac{-x^2}{(1 - x^2)^{1/2}} + (1 - x^2)^{1/2}$       18)  $\frac{-4x(-2 - 2x^2)}{(x^2 - 1)^3} - \frac{4x}{(x^2 - 1)^2}$

## VIII. Solving Equations

Solve for x.

1)  $x^2 + x - 1 = 11$       2)  $12x^2 - 24x = 0$       3)  $160x - 16x^2 = 256$       4)  $2x^3 - 128x = 0$

5)  $3x^2 - 2x - 2 = -1$       6)  $2\sin x \cos x - \sin x = 0 \quad 0 < x < 2\pi$       7)  $27 - 3x^2 = 0$

8)  $6x(x-4) + 6(x-4)^2 = 0$       9)  $(x+5)^2 + 2(x-3)(x+5) = 0$       10)  $\frac{2x(x^2-4) - 2x^3}{(x^2-4)^2} = 0$

11)  $\frac{4(x^2+1) - (4x)(2x)}{(x^2+1)^2} = 0$       12)  $\frac{-2x(x^2+1)^2 - 4x(1-x^2)(x^2+1)}{(x^2+1)^4} = 0$

Solve for z.

13)  $2x + 3xz + 3y + 3y^2z = 0$       14)  $2x + 18yz - 4 + 3z = 0$

15)  $2yz = (x-y)(2x+z) + (x^2+y)(1-z)$       16)  $xz \cos y + \sin y = -y \sin x + z \cos x$

## IX. The Natural Logarithm and e

Expand the Logarithm.

1)  $\ln x^2 y^3$       2)  $\ln \frac{x^5}{y^4}$

Condense the Logarithm.

3)  $4 \ln x + 10 \ln y - 3 \ln z$

Solve for x.

4)  $e^{\ln x} = 8$       5)  $\ln e^x = 10$       6)  $\ln x = 6$       7)  $e^x = 12$       8)  $12 - 3e^x = -6$

9)  $\ln \sqrt{x+5} = 1$       10)  $\ln 7x = 3$       11)  $\ln x - \ln 3 = \ln 4$

Students should know the following properties:

a)  $\ln e^x = x$       b)  $e^{\ln x} = x$       c)  $\ln 1 = 0$

## X. Horizontal and Vertical Asymptotes

State any and all horizontal and vertical asymptotes for each.

1)  $y = 6x^2 - 8x + 1$

2)  $y = \frac{3}{7x^2 - x}$

3)  $y = \frac{x^2 - 10x + 21}{x^2 + 7x + 6}$

4)  $y = \frac{5 - 2x^2}{x^2 - 4}$

5)  $y = \frac{x^3 + 7x^2 - 5}{3x^2 + 3x - 216}$

6) Given that  $f(x) = \frac{1}{x - 52}$  and  $g(x) = x^2 + 16$ , state the vertical and horizontal asymptotes for  $f(g(x))$ .

## XI. Even and Odd Functions

1) State whether the function is even, odd, or neither.

a)  $f(x) = 3x^2 + 7x^4 + 5$

b)  $f(x) = x^2 + 6x - 3$

c)  $f(x) = \frac{x}{x^2 + 8}$

2) If  $f(x)$  is even and  $f(3) = 5$ , what is  $f(-3)$  ?

If the slope of  $f(x)$  at  $x = 3$  is 7, what is the slope at  $x = -3$  ?

3) If  $f(x)$  is odd and  $f(-6) = -10$ , what is  $f(6)$  ?

If the slope of  $f(x)$  at  $x = -6$ , what is the slope at  $x = 6$  ?

## XII. Exponents

1) Rewrite the following radicals using exponents.

a)  $\sqrt{x}$

b)  $\sqrt[3]{x}$

c)  $\sqrt[5]{x^3}$

2) Rewrite the following using negative exponents.

a)  $\frac{1}{x^4}$

b)  $\frac{2}{x^{10}}$

c)  $\frac{3}{4x^6}$

d)  $\frac{1}{\sqrt{x}}$

e)  $\frac{3}{\sqrt[4]{x}}$

f)  $\frac{2}{7\sqrt[8]{x}}$

g)  $\frac{1}{\sqrt{x+2}}$

4) Simplify the following.

a)  $(3x^2)(2x^3)$

b)  $(-5x^4)^2$

c)  $(5x^3 + 7x^2 - x + 2)x^{-2}$

d)  $\frac{3x^5 - 7x^2 + 6x - 1}{x}$

5) Rewrite and simplify the following using positive exponents.

a)  $5x^{-2/3}$

b)  $7x^{-1/2}$

c)  $10(x+1)^{-1/5}$

d)  $\frac{2}{3}(9-x^5)^{-1/3}(-2x)$

e)  $\frac{1}{3}(9x^2+4)^{-3/4}(18x)$

### XIII. Inverses

1) If the points (3,2) and (-5,7) are on the graph of  $f(x)$  and  $g(x) = f^{-1}(x)$ , state two points that are on the graph of  $g(x)$ .

2) Find the inverse.

a)  $f(x) = 3x + 5$

b)  $g(x) = x^2 - 10$

Students should know that if two functions are inverses:

1) Their graphs are reflections about the line  $y = x$ . 2) Both  $f(g(x)) = x$  and  $g(f(x)) = x$

### XIV. Complex Fractions

Simplify the following.

1)  $\frac{\frac{1}{x+5} - \frac{1}{5}}{x}$

2)  $\frac{\frac{1}{(x+2)^2} - \frac{3}{x+2}}{x^2}$

3)  $\frac{\frac{1}{\sqrt{x-3}} + 7\sqrt{x-3}}{(x-3)^{3/2}}$

### XV. Trigonometry

Simplify.

1)  $\cos \theta \tan \theta$

2)  $\frac{\cos x}{\cot x}$

3)  $\frac{-4\sin(5x)}{\cos(5x)}$

Students should understand the following:

1)  $(\cos x)(\cos x) = \cos^2 x$  or  $(\cos x)^2$  ...It is not the same as  $\cos x^2$

2)  $\sin^5 x = (\sin x)^5$  .... not  $\sin x^5$

3)  $\sin^{-1} x = \arcsin x$  ... not  $\frac{1}{\sin x}$

Students MUST know the Unit Circle!!!