Lesson Study Resource Kit

Laying the Groundwork: Introducing Students to Inverse Operations

A CPALMS Lesson Study Resource Kit for Math Grades K-2

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Introduction

This K-2 mathematics resource kit is designed to assist teams of teachers engaged in lesson study with essential background information and exemplary lesson resources to construct an instructional unit that prepares students for achieving an understanding of inverse operations that corresponds to the Common Core State Standards for Mathematics. Included in this resource kit are:

- Background research on the importance of preparing elementary students for understanding the concept of inverse operations,
- Formative assessment tasks related to this topic,
- A "Developmental Story" that illustrates a learning progression for putting students on a path to achieving adult learning expectations of this concept,
- An adaptable model lesson for teaching the concept of inverse operations to students in K-2, and
- Examples of student responses to problems posed in the exemplary lesson.

What is Lesson Study?

Lesson study is a process that engages small collaborative teams of teachers in the selection, modification, teaching, and analysis of research lessons that serve as vehicles for professional learning. When viewed from this perspective, lesson study is teacher-led, embedded in content, active, hands-on, and focused on student learning outcomes (Lewis & Hurd, 2011; Stepanik, et al., 2007). In short, lesson study is a professional development framework that:

- Values teaching, teachers, and the professional teaching community,
- Provides an important learning structure—the research lesson,
- Values the long-term learning and development of students,
- Builds a shared knowledge base (Lewis & Hurd, 2011, p. 6), and
- Motivates teachers to collaborate in examining their practice through the lens of student learning.

Why do Lesson Study?

Lesson study provides teachers with opportunities to work collectively to increase their proficiency in:

- analyzing and using student data in planning for instruction,
- reviewing the relevant research on the teaching and learning of a subject, topic, or standard,
- selecting and/or developing standards-based research lessons and formative assessment tasks that probe student understanding, and
• assessing the effectiveness of instruction in achieving desired student learning outcomes.
How to Use Lesson Study Resource Kits in CPALMS Lesson Study Support System (LSSS)

The process outlined in CPALMS’ LSSS breaks the lesson study cycle into seven distinct steps:

Step 1: Establish your Lesson Study Team and Schedule Planning Time
Step 2: Set your Team’s Goals
Step 3: Conduct Background Research
Step 4: Plan your Team’s Lesson
Step 5: Teach your Team’s Lesson
Step 6: Debrief and Reflect

The sequence of steps and sub-steps in the LSSS was organized to support lesson study by both novice and experienced lesson study teams. If you and your team are new to lesson study, the LSSS provides a systematic set of instructions and prompts that are designed to guide you through your first lesson study cycle. In order to optimize the professional learning opportunities that lesson study provides, it is highly recommended that novice groups adhere to the directions outlined in the steps. If you and your team are more familiar with lesson study, the step-wise approach outlined in the LSSS provides a framework for engaging in lesson study at a level of depth that research has shown to be effective in enhancing teacher content knowledge as well as student learning.

In order to assist your team in this process, the LSSS combines important background information that directs your team to essential resources, tips, and tools that require direct action on your team’s part.

**STEP 1: Establish your Lesson Study Team and Schedule Planning Time**

At its most effective, lesson study engages small teams of teachers in collaborative professional learning that is focused on improving student learning outcomes. Because it is an in-depth approach that requires teachers to reach consensus about the goals and conduct of the team’s activities, it is recommended that teams be composed of teachers who share an interest in engaging in collaborative reflective practice and have ample time to participate fully.

**Step 1, Task 1: Establish Your Lesson Study Team**

Once you have identified prospective team members for a lesson study cycle, go to “Establish Your Lesson Study Team,” in task 1 of Step 1 in the LSSS, and follow the instructions for creating “member roles” in the instructions panel. Once you have identified what member roles you desire for your team, invite an individual team member by inserting his or her name in the “Find a member” dialogue box in the “Add Member” section at the bottom of the page. Once a new member’s name appears, you will be prompted to assign roles and access to the
LSSS for that member. If you want each member to have “write access” to the lesson study, you can check the “Give write access” box in the permissions and role section. Once you have entered this information, click “Invite” to the left of the member’s name, and an email will be sent to the email address provided by the teacher in the CPALMS database.

Step 1, Task 2: Orient your Team to Lesson Study
Once you have assembled a team it is important to ensure that a shared understanding of lesson study is developed among its members. Typically, the first meeting of a new team is devoted primarily to orienting everyone to the goals and protocols that characterize effective lesson study. One strategy for orienting your team to lesson study is to assign a short reading about lesson study to team members in advance of the first meeting so that they can have an opportunity to share their impressions of lesson study. If you do not have access to any literature on lesson study, choose one or more readings from among the Lesson Study Resource Links located in the “Instructions” Panel in the “Orient to Lesson Study” task in step 1 of the LSSS.

Step 1, Task 3: Develop Group Norms
Group norms constitute the ground rules that govern the interactions of lesson study team members throughout all phases of the cycle. Lewis & Hurd (2011) recommend that lesson study teams reflect upon the qualities that have helped and hindered professional learning collaborations in previous experiences.

Examine the Sample Group Norms located in the Instructions Panel on the “Develop Group Norms” task, Step 1, Task 3 in the LSSS.

Create norms for your team members by selecting from among the sample group norms and/or creating your own, in the Add/Edit Group norms feature on the “Develop Group Norms” page.

Step 1, Task 4: Schedule Planning Time
Once your team has assigned roles for each member, a meeting schedule can be created that provides the space and time for all projected lesson study team meetings and activities. More information and suggestions for scheduling meetings can be found on pages 38 – 45 of the Florida Bureau of School Improvement’s A Guide to Implementing Lesson Study (http://www.scribd.com/doc/95555469/A-Guide-to-Implementing-Lesson-Study).

The “Schedule Planning Time” tool is located in the fourth task in step 1 on the LSSS. This page includes a calendar for scheduling meetings that includes features for generating the meeting itinerary and affixing notes for team members.
STEP 2: Set Your Team’s Goals

Goal setting is a central activity in lesson study. From the establishment of an over-arching research theme, to the subsequent development of a unit plan, and research lesson, goal setting is essential to laying the groundwork for a coherent “teaching-learning plan” (Lewis, 2011, p. 47) that will guide your team’s efforts.

Figure 2. Teaching and Learning Plan (Lewis & Hurd, 2011, p. 49)

The illustration in Figure 2 depicts the three essential elements of the teaching-learning plan as nested circles, with the research lesson as part of a larger unit plan, and both embedded within an over-arching research theme. As your team progresses through the planning of the research lesson, it should continuously check that the goals of these three elements are aligned.

Step 2, Task 1: Develop a Research Theme

The research theme represents a long-term goal that your team has for your students. The role of the research theme in lesson study is to orient your team’s activities toward an overarching mission that directs your lesson study activities. In that sense, a research theme is important in setting the tone for your team’s efforts for one or more lesson study cycles. When determining your team’s research theme, it is helpful to consider your school’s mission statement as well as the particular social and academic qualities that your team would like for your students to develop. Examples of research themes include:

- Our goal is to develop science learners who are engaged and able to apply evidence-based reasoning to scientific questions.
- We want our students to develop confidence in themselves and demonstrate persistence when faced with challenging mathematical problems.
In the first example, two broad outcomes are articulated. The development of “engaged” science learners is a desired social outcome, while the application of “evidence-based reasoning” represents an academic objective. The second example focuses on personal qualities of students that are related to improved learning of mathematical material. The Research Theme tool will guide you through the process of selecting a research theme.

Use the “Develop a Research Theme” tool, Step 2, Task 1, in the LSSS to:
- Enter your school’s mission statement
- Create lists of desired and undesired student qualities your team wants to address through lesson study
- Bridge the gap between those desired and undesired student qualities that you and your team plan to address in the LS cycle, and
- Record your team’s “Research Theme” for the LS cycle.

Step 2, Task 2: Relevant State Standards

| MACC.K.OA.1.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. |
| MACC.K.OA.1.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. |
| MACC.1.OA.1.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |
| MACC.2.OA.1.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |

Step 3: Conduct Background Research

Once your team has identified a research focus around which it will center its inquiry, the work of curriculum study can begin in earnest. The importance of conducting background research cannot be overstated. It is at this stage of the lesson study cycle that teachers have the opportunity to enhance both their content and pedagogical knowledge in a collaborative and collegial fashion. The LSSS breaks this all-important step in the process into five separate tasks:

- Explore Research and Resources on Teaching and Learning
- Develop a Thorough Understanding of Related Benchmarks and Standards
- Explore Formative Assessments
• Examine Informational Texts, and  
• Study Curriculum

Although a single LS cycle may be focused primarily on one of these tasks, teams have the option of incorporating any that they would like to in a given cycle.

**Step 3, Task 1: Explore Research and Resources on Teaching and Learning**

The purpose of this step is to deepen your understanding of the issues related to teaching and learning of the topics and concepts that underpin your research focus.

The following reading has been selected to help you better understand some of the pedagogical issues underlying your selected research focus. Plan to read each one on your own prior to your next team meeting. Questions for discussion are included for your team to consider at your next team meeting. Depending on your time constraints, you may choose to address one reading per meeting for the next few meetings or address all of them at one meeting.


**Research Summary:**

This article focuses on the importance of laying the groundwork early in preparing students for achieving sophisticated understandings of the concept of inverse operations. The article argues that teachers must be explicit in calling attention to the fact that inverse involves reversing the operation for which it is associated, and that a working understanding of the concept of inverse operations should be presented as a powerful tool for solving problems for students throughout K-12. The most relevant parts of the article for teachers of K-2 students can be found on pages 108-110.

**Guiding Questions**

- How does addition undo subtraction?
- How does subtraction undo addition?
- How do different textbooks define inverse operations?
- How is inverse addressed at each grade level?

**Step 3, Task 2: Develop a Thorough Understanding of Related Benchmarks and Standards**

Before creating your unit plan and research lesson for your team’s lesson study cycle, it is important to develop a thorough understanding of any standards or benchmarks that are relevant to your research focus. The “Unpack Related Benchmarks/Standards” tool located in Task 2 of Step 3 of the LSSS, provides your team with a means of:

- exploring any misconceptions or preconceptions that are associated with an individual standard
- identifying the content expectations for each standard/benchmark, and
• formulating essential questions and/or learning goals for each standard

Open the “Unpack Related Benchmarks/Standards” tool and address the dialogue boxes for each of the three items in the list above for each selected standard or benchmark that pertains to your team’s research focus.

As individuals, read the following text regarding the use of the mathematical practice standards, before discussing it as a team.

• How can your team design your unit so that it makes optimal use of the CCSS’ mathematical practices?

Step 3, Task 3: Explore Formative Assessments
Before your team plans its unit and research lesson, it is helpful to examine assessment items and tasks associated with your team’s research focus, in order to identify possible formative assessment tasks that can be incorporated into your team’s unit planning. Your team is encouraged to read through the next four formative assessment tasks and consider whether any of them are suitable for your students in this lesson study cycle. You may also design your own formative assessment task, if you have experience in the design and use of them. The following MFAS formative assessment tasks have been included in this resource kit for your team to explore:

1. **GENERAL INFORMATION AND SUMMARY**

   **Title:** Inverse Operations
   **Resource ID:** 12820
   **Description:** Students are asked to determine and write addition and subtraction fact families and explain their reasoning.
   **Subject(s):** NGSSS: Mathematics
   **Grade Level(s):** 1
   **Intended Audience:** Educators
   **Special Materials Needed:**
   - Paper and pencil
   - Connecting cubes (at least 15)
   **Freely Available:** Yes
   **Instructional Component Type(s):** Formative Assessment (Primary Type)
   **Resource Collection:** MFAS - NGSSS Mathematics

   [RELATED STANDARDS (1)]
MA.1.A.1.2: Identify, describe, and apply addition and subtraction as inverse operations.

Cognitive Complexity: Level 2: Basic Application of Skills & Concepts

Date Adopted or Revised: 09/07

Belongs to: BIG IDEA 1

Related Instructional Resources » More Information »

FORMATIVE ASSESSMENT TASK

Instructions for Implementing the Task

1. The teacher shows the student one addition sentence and asks the student to provide the three related addition or subtraction facts using the same three numbers. The teacher may use this number sentence as an example:
   $$3 + 4 = 7.$$  
The student should respond $$4 + 3 = 7, \ 7 - 3 = 4 \text{ and } 7 - 4 = 3.$$  

2. The teacher provides the student with the following three sets of three numbers. The teacher asks the student to make the related facts for those numbers and explain his or her answer for each set.
   - A. 5, 7, 12
   - B. 9, 3, 6
   - C. 5, 10, 5

3. The teacher provides the student with each set of three numbers listed below and asks the student to determine whether or not the numbers would make related facts. The teacher asks the student to explain his or her reasoning.
   - A. 4, 5, 9
   - B. 6, 10, 3
   - C. 5, 3, 2
   - D. 7, 2, 6

Vocabulary

Associated with the Assessment

addition, subtraction, related facts, inverse operations

Task Rubric

Level I - Getting Started
Misconception / Error
The student understands that a related fact has two addition and two subtraction sentences but uses different numbers.

Examples of Student Work at this Level

Problem 2A
5 + 7 = 12
12 + 7 = 19
7 - 5 = 2
12 - 7 = 5

Problem 2B
6 + 3 = 9
9 + 3 = 12
6 - 3 = 3
9 - 3 = 6

Problem 2C
10 + 5 = 15
5 + 5 = 10
10 - 5 = 5
5 - 10 = 5

Questions Eliciting Thinking
What do you notice about the related facts we did together?

Instructional Implications
Give students experiences working with related facts; make sure to explain that each related fact has three numbers that are used for both the addition and subtraction sentences.

Level II – Moving Forward

Misconception / Error
The student can only create the addition portion of each fact family.

Examples of Student Work at this Level

Problem 1
4 + 3 = 7
3 + 4 = 7

Problem 2A
5 + 7 = 12
7 + 5 = 12

Problem 2B
6 + 3 = 9
3 + 6 = 9

Problem 2C
5 + 5 = 10

Questions Eliciting Thinking
What would the subtraction related facts be?

Instructional Implications
Have the student practice with manipulatives to show that addition and subtraction are inverse operations.
Problem 3A
4 + 5 = 9
5 + 4 = 9

Problem 3B
No addition portions for this fact family

Problem 3C
3 + 2 = 5
2 + 3 = 5

Problem 3D
No addition portions for this fact family

Level III – Almost There

**Misconception / Error**
The student tries to make the numbers work together by writing the incorrect number, even when he or she knows that it is not the correct answer.

**Examples of Student Work at this Level**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>3B</td>
<td>6 + 3 = 10</td>
</tr>
<tr>
<td></td>
<td>3 + 6 = 10</td>
</tr>
<tr>
<td></td>
<td>10 - 3 = 6</td>
</tr>
<tr>
<td></td>
<td>10 - 6 = 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>6 + 2 = 7</td>
</tr>
<tr>
<td></td>
<td>2 + 6 = 7</td>
</tr>
<tr>
<td></td>
<td>7 - 2 = 6</td>
</tr>
<tr>
<td></td>
<td>7 - 6 = 2</td>
</tr>
</tbody>
</table>

The student can create related facts for Parts 1 and 2 but cannot determine which sets of

**Questions Eliciting Thinking**

- Problem 3B
  - Show me 6 + 3. How did you decide it was 10?
  - Let’s read the directions for this part of the task again. Reread student directions. Does that set of numbers make related facts? Why not?

**Instructional Implications**

- Have the student practice with manipulatives to show that addition and subtraction are inverse operations.
numbers create related facts in Part 3 of the task.

Level IV – Got It

<table>
<thead>
<tr>
<th>Misconception / Error</th>
<th>Examples of Student Work at this Level</th>
<th>Questions Eliciting Thinking</th>
<th>Instructional Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are no misconceptions or errors.</td>
<td>The student correctly creates the related facts for all parts of this task.</td>
<td>Can you write a set of numbers that would create related facts?</td>
<td>Give students practice in creating their own related facts.</td>
</tr>
</tbody>
</table>

2. GENERAL INFORMATION AND SUMMARY

Title: Inverse Operations with Manipulatives

Resource ID: 12821

Description: Students are given several addition and subtraction problems and asked for the related facts (inverse).

Subject(s): NGSSS: Mathematics

Grade Level(s): 1

Intended Audience: Educators

Special Materials Needed: At least 16 connecting cubes in 2 colors (or another manipulative in 2 colors that students can use to count)

Freely Available: Yes

Instructional Component Type(s): Formative Assessment (Primary Type)

Resource Collection: MFAS - NGSSS Mathematics

 RELATED STANDARDS (1)
MA.1.A.1.2: Identify, describe, and apply addition and subtraction as inverse operations.
Cognitive Complexity: Level 2: Basic Application of Skills & Concepts  
Date Adopted or Revised: 09/07
Belongs to: BIG IDEA 1
Related Instructional Resources »  More Information »

FORMATIVE ASSESSMENT TASK

Instructions for Implementing the Task
The teacher has connecting cubes in two colors available (at least 16). The teacher shows the student an equation and asks him or her for a related fact with the inverse operation. The teacher observes the student working on the problem, asks the student to explain his or her answer and reasoning after each problem, and listens to the student's thinking process.

Problem A. The teacher shows $2 + 3 = 5$. The teacher says, “Write a subtraction equation that uses the same three numbers as this addition equation.”

Problem B. The teacher shows $4 + 2 = 6$. The teacher says, “Write a subtraction equation that uses the same three numbers as this addition equation.”

Problem C. The teacher shows $6 + 3 = 9$. The teacher says, “Write a subtraction equation that uses the same three numbers as this addition equation.”

Problem D. The teacher shows $10 - 4 = 6$. The teacher says, “Write an addition equation that uses the same three numbers as this subtraction equation.”

Problem E. The teacher shows $8 - 5 = 3$. The teacher says, “Write an addition equation that uses the same three numbers as this subtraction equation.”

Problem F. The teacher shows $7 - 2 = 5$. The teacher says, “Write an addition equation that uses the same three numbers as this subtraction equation.”

Vocabulary Associated with the task: addition, subtraction, related facts, inverse operation
### Level I - Getting Started

**Misconception / Error**
The student only changes the operation from the related fact given.

**Examples of Student Work at this Level**
- $3 - 2 = 1$ or $2 - 3 = 1$
- $4 - 2 = 2$
- $6 - 3 = 3$
- $10 + 4 = 14$
- $8 + 5 = 13$
- $7 + 2 = 9$

**Questions Eliciting Thinking**
Which one is a related fact for $8 + 3 = 11$?
- a. $8 - 3 = 5$, or
- b. $11 - 3 = 8$?

**Instructional Implications**
Give student experience with related facts.

Make sure to explain that each related fact has three numbers that are used for both the addition and subtraction problems.

### Level II – Moving Forward

**Misconception / Error**
The student can only create the addition portion of each fact family.

**Examples of Student Work at this Level**
- $3 - 2 = 1$ or $2 - 3 = 1$
- $4 - 2 = 2$
- $6 - 3 = 3$
- $6 + 4 = 10$ or $4 + 6 = 10$
- $5 + 3 = 8$ or $3 + 5 = 8$
- $5 + 2 = 7$ or $2 + 5 = 7$

**Questions Eliciting Thinking**
$2 + 3 = 5$. Show me this with manipulatives.

How can you show a subtraction problem using the same numbers?

**Instructional Implications**
Have the student practice with manipulatives to show that addition and subtraction are inverse operations.

### Level III – Almost There
### Level IV – Got It

<table>
<thead>
<tr>
<th>Misconception / Error</th>
<th>Examples of Student Work at this Level</th>
<th>Questions Eliciting Thinking</th>
<th>Instructional Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are no misconceptions or errors.</td>
<td>The student correctly creates the related facts for all parts of this task.</td>
<td>Why is it impossible to create related facts when adding three numbers?</td>
<td>Give student practice in creating his or her own related facts.</td>
</tr>
</tbody>
</table>

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**Title:** Using Inverse Operations

**Resource ID:** 36517

**Description:** Students identify an addition equation that can be used to solve a subtraction problem.

**Subject(s):** CCSS: Mathematics

**Grade Level(s):** 1

**Intended Audience:** Educators

**Special Materials Needed:** Using Inverse Operations worksheet

**Freely Available:** Yes

**Instructional Formative Assessment (Primary Type):** Formative Assessment
3. GENERAL INFORMATION AND SUMMARY

- RELATED STANDARDS (1)

» MACC.1.OA.2.4: Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

Cognitive Complexity: Level 2: Basic Application of Skills & Concepts

| Date Adopted or Revised: 10/10 |
| Belongs to: Understand and apply properties of operations and the relationship between addition and subtraction. |

Related Instructional Resources » More Information »

FORMATIVE ASSESSMENT TASK

Instructions for Implementing the Task

This task can be completed individually or in small groups.

1. The teacher provides the student with the Using Inverse Operations worksheet.

2. The teacher reads aloud the problem at the top of the worksheet. Then, the teacher asks the student whether the student would use addition or subtraction to solve the problem and to write an equation that could be used to solve the problem.

Note: Some students may not know the term equation and may instead know it as a number sentence. Students may need help determining
where to write the numbers and operational sign.

The teacher then prompts the student to circle the addition fact that would help solve the subtraction problem. After the student chooses the addition fact, the teacher should have the student explain his or her answer.

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**Task Rubric**

**Level I - Getting Started**

The student does not demonstrate an understanding of subtraction as an unknown-addend problem. **Misconception / Error**

**Examples of Student Work at this Level**
The student can generate the equation $13 - 4 = \_\_\_$, but is unable to identify a related addition equation that can be used to solve $13 - 4 = \_\_\_$.  

**Questions Eliciting Thinking**
Do you know what $13 - 4$ equals?  
So, if $13 - 4 = 9$, then what number can you add to four to get 13?  
So, you can add 9 to 4 to get 13. Can you write this as an addition equation? (Guide the student to write $4 + 9 = 13$.)

If $3 + 5 = 8$, then what should $8 - 5$ equal?  
If you had to solve the equation, $7 - 3 = \_\_\_$, what addition fact could you use?

**Instructional Implications**
Model related addition and subtraction problems with cubes. For example, start with five cubes and take away three leaving two. Then, show how adding back the same three cubes yields the original amount of five. Relate the model to equations.

Use a number line to show the relationship between addition and subtraction (e.g., the difference between three and five is the same as the amount that must be added to three to make five).
Have the student write related addition and subtraction equations (e.g., $6 - 2 = 4$ and $4 + 2 = 6$). Focus on the relationship between the problems rather than simply moving numbers to fit the equations.

Present the Counting On To strategy as a way to use addition to solve subtraction problems.

Level II – Moving Forward

**Misconception / Error**
The student does not understand how to use a related addition equation to solve a subtraction problem.

**Examples of Student Work at this Level**
The student can generate the equation $13 - 4 = \underline{\text{____}}$, and can identify the related addition equation, but cannot use the equation to find the answer to $13 - 4 = \underline{\text{____}}$.

**Questions Eliciting Thinking**
What added to four makes 13?

How can we use that to help us know the addition problem that relates to the subtraction problem of $13 - 4$?

If $3 + 5 = 8$, then what should $8 - 5$ equal?

If you had to solve the equation, $7 - 3 =$

**Instructional Implications**
Provide the student opportunities to see how addition and subtraction are related.

Use a number line to show the relationship between addition and subtraction (e.g., the difference between three and five is the same as the amount that must be added to
____, what addition fact could you use? three to make five).

Have the student solve related addition and subtraction problems and then explain how the two sets of problems are alike.

Have the student write related addition and subtraction equations (e.g., $6 - 2 = 4$ and $4 + 2 = 6$).

Present the Counting On To strategy as a way to use addition to solve subtraction problems.

Level III – Almost There

**Misconception / Error**
The student cannot explain how a related addition equation can be used to solve a subtraction problem.

**Examples of Student Work at this Level**
The student can generate the equation $13 - 4 = ____$, identify the related addition equation, and use it to find the answer to $13 - 4 = ____$. However, the student is unable to justify or explain his or her choice of equation or answer.

**Questions Eliciting Thinking**
How do you know that $9 + 4 = 13$ is the addition equation that helps us solve $13 - 4 = ____$?

What do we know about the numbers 9, 4, and 13? How are addition and subtraction related?

Do you think we can always use addition to solve subtraction problems?

**Instructional Implications**
Have the student solve related addition and subtraction problems, and then explain how the two sets of problems are alike.

Have the student write related addition and subtraction equations (e.g., $6 - 2 = 4$ and $4 + 2 = 6$), and explain the relationship between
Allow other students in the class to share their thinking about how to solve subtraction problems using addition strategies.

### Level IV – Got It

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>The student has no misconceptions or errors.</td>
<td>The student solves the equation $13 - 4 = _____$ by identifying a related addition equation, $9 + 4 = 13$, and explains that, consequently, $13 - 4 = 9$. The student is then able to explain that addition and subtraction are inverse operations.</td>
<td>How are addition and subtraction related? Do you think we can always use addition to solve subtraction problems? Why?</td>
<td>Allow the student opportunities to generalize this relationship. Encourage the student to try to do so without using numbers, and rather to simply describe the way each operation works.</td>
</tr>
</tbody>
</table>

Note: It is not necessary for students to say "inverse operations." They may instead say that the two operations are opposites or that they undo each other.

Provide the student with a subtraction problem that has larger numbers (e.g., $37 - 29 = \_\_\_\_$) to determine if he or she can generate the addition equation that could be used to solve this problem.

Have the student create his or her own problems where inverse operations
4. **GENERAL INFORMATION AND SUMMARY**

**Title:** Use Addition to Solve Subtraction  
**Resource ID:** 36582  
**Description:** Students are given a subtraction problem and asked to solve the problem using a related addition fact.  
**Subject(s):** CCSS: Mathematics  
**Grade Level(s):** 1  
**Intended Audience:** Educators  
**Special Materials Needed:** Use Addition to Solve Subtraction worksheet  
**Freely Available:** Yes  
**Instructional Component Type(s):** Formative Assessment (Primary Type)  
**Attachments:** MFAS_UseAdditionToSolveSubtraction_Worksheet.docx  
**Resource Collection:** MFAS Common Core

**RELATED STANDARDS (1)**

» **MACC.1.OA.2.4:** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
   
   Cognitive Complexity: Level 2: Basic Application of Skills & Concepts  
   Date Adopted or Revised: 10/10  
   Belongs to: Understand and apply properties of operations and the relationship between addition and subtraction.  
   Related Instructional Resources »  
   More Information »

**FORMATIVE ASSESSMENT TASK**
Instructions for Implementing the Task

This task can be completed individually or in small groups.

The teacher gives the student the *Use Addition to Solve Subtraction* worksheet.

The teacher asks the student, “What do we know about addition and subtraction?”

The teacher asks the student, “Is there a related addition fact that would help you solve the problem $10 - 7 = ______$?” If the student tries to subtract instead of using inverse operations to solve the problem, the teacher prompts the student by asking, “Seven plus what equals 10?” The teacher waits for the student to respond. If the student responds correctly, the teacher asks the student how he or she might use $7 + 3 = 10$ to solve the equation $10 - 7 = ______$. Then, the teacher asks the student to solve $10 - 7 = ______$.

Note: If the student is successful with step 3, the teacher proceeds to step 4. If the student continues to struggle and wants to subtract to find the answer, the teacher should stop the task and refer to Level I of the rubric.

The teacher prompts the student to use a related addition fact to solve the problem $9 - 3 = ______$.

Note: The teacher should observe if the student is able to say $6 + 3 = 9$ so $9 - 3 = 6$ without any further prompting.

 Task Rubric

Level I - Getting Started

<table>
<thead>
<tr>
<th>Misconception / Error</th>
<th>Examples of Student Work at this Level</th>
<th>Questions Eliciting Thinking</th>
<th>Instructional Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student does not demonstrate an understanding</td>
<td>The student uses a related addition fact</td>
<td>You know that $10 - 7$</td>
<td>Model related</td>
</tr>
</tbody>
</table>
understanding of subtraction as an unknown-addend problem.

strategy such as Counting Down to solve the subtraction problem (10 – 7 = ____), and knows that the answer is three; however, the student cannot identify the corresponding addition equation, even with prompting.

= 3. So, what added to seven makes 10? (Write out both 10 – 7 = 3 and 7 + ____ = 10 for the student so that he or she can see the equations.)

If 9 – 3 = 6, what added to six makes nine? (Write out both 9 – 3 = 6 and 6 + ____ = 9 for the student so that he or she can see the equations.)

Can you see a relationship between these pairs of equations?

Use a number line to show the relationship between addition and subtraction (e.g., the difference between 7 and 10 is the same as the amount that must be added to seven to make 10.)

Have the student write related addition and subtraction equations (e.g., 6 – 2 = 4 and 4 + 2 = 6).

Level II – Moving Forward

<table>
<thead>
<tr>
<th>Misconception / Error</th>
<th>Examples of Student Work at this Level</th>
<th>Questions Eliciting Thinking</th>
<th>Instructional Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student can identify an addition problem related to a given subtraction</td>
<td>Initially, the student says that 10 + 7 will help solve the</td>
<td>What added to seven makes 10? How can we use that to help</td>
<td>Provide the student opportunities to see how addition and</td>
</tr>
</tbody>
</table>
problem, but only with significant prompting from the teacher, and even so, is not always successful.

With prompting, the student determines that if $7 + 3 = 10$, then $10 - 7 = 3$. But, given the problem $9 - 3 = ____$, the student says that the addition fact $9 + 3$ would help find $9 - 3$.

The student is unable to identify the addition equation that can be used to solve $9 - 3 = ____$, even with prompting from the teacher.

us know the addition problem that relates to the subtraction problem of $10 - 7$?

What is $10 - 7$? What addition facts go with that problem?

If $9 - 3 = 6$, what added to six makes nine? (Write out both $9 - 3 = 6$ and $6 + ____ = 9$ for the student so that he or she can see the equations.)

Can you see a relationship between these pairs of equations?

What do you know about addition and subtraction? How are they related?

subtraction are related.

Use a number line to show the relationship between addition and subtraction (e.g., the difference between 7 and 10 is the same as the amount that must be added to seven to make 10).

Have the student solve related addition and subtraction problems and then discuss how the two sets of problems are alike.

Have the student write related addition and subtraction equations (e.g., $6 - 2 = 4$ and $4 + 2 = 6$).

Present the Counting On To strategy in the context of Compare (Result Unknown) subtraction problems as a way to use addition to solve subtraction problems.
## Level III – Almost There

### Misconception / Error
Some guidance from the teacher is needed in order for the student to identify an addition problem related to a given subtraction problem.

### Examples of Student Work at this Level
With guidance, the student can identify a corresponding addition equation, and can use it to solve a given subtraction equation.

The student is able to identify the addition equation that can be used to solve $10 - 7 = \_\_\_$ with some guidance from the teacher.

The student is able to identify the addition equation that can be used to solve $9 - 3 = \_\_\_$ with some guidance from the teacher.

### Questions Eliciting Thinking
Why is $7 + 3 = \_\_\_$ the addition problem that helps us find $10 - 7$?

What do we know about the numbers $10$, $7$, and $3$?

How are addition and subtraction related?

Will this always work with subtraction problems? Can you find an addition fact that goes with any subtraction problem?

### Instructional Implications
Provide the student with subtraction problems and ask him or her to write the related addition problem. Focus on the relationship between the problems rather than simply moving numbers to fit the equations.

Present the Counting On To strategy in the context of Compare (Result Unknown) subtraction problems as a way to use addition to solve subtraction problems.

Allow other students in the class to share their thinking about how to solve subtraction problems using addition strategies.
### Level IV – Got It

<table>
<thead>
<tr>
<th>Misconception / Error</th>
<th>Examples of Student Work at this Level</th>
<th>Questions Eliciting Thinking</th>
<th>Instructional Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student has no misconceptions or errors.</td>
<td>In response to the second problem, the student says $6 + 3 = 9$ so $9 - 3 = 6$ without any further prompting.</td>
<td>How do you know that $7 + 3 = 10$ is the addition problem that helps us solve $10 - 7$? Do you think we can always use addition to solve subtraction problems? Why?</td>
<td>Allow the student opportunities to generalize this relationship. Encourage the student to try to do so without using numbers and rather to simply describe the way each operation works.</td>
</tr>
<tr>
<td></td>
<td>The student is able to describe, in general, how pairs of addition and subtraction problems are related.</td>
<td></td>
<td>Provide the student with a subtraction problem that has larger numbers (e.g., $37 - 29 = ____$) to determine if he or she can generate the addition equation that could be used to solve this problem.</td>
</tr>
<tr>
<td></td>
<td>The student can explain the inverse relationship between addition and subtraction.</td>
<td></td>
<td>Have the student create his or her own problems where inverse operations can be used as a strategy for solving.</td>
</tr>
</tbody>
</table>

### Step 3, Task 4: Examine State, National, and International Assessment Items

In this task, your team can review released assessment items from large-scale assessments including, FCAT 2.0, EOCs, the National Assessment of Educational Progress (NAEP), Trends in International Math and Science Study (TIMSS), and the Program for International Science
Achievement (PISA). Through discussion of the central performance tasks embedded in these assessment items, you and your team can work toward identifying key elements of an assessment plan for the instructional unit that is being developed. The assessment items that follow have been selected for your team’s consideration.

**Step 3, Task 5: Examine Informational Texts**

Anchor standard 10 of the *Common Core State Standards for English Language Arts & Literacy in History, Social Studies, Science, and Technical Subjects* (2010), calls for students to:

Read and comprehend complex literary and informational texts independently and proficiently (p.10). The success of Florida’s students in achieving proficiency in this standard depends, among other things, upon the ability of teachers in all subject areas to *identify, evaluate, and integrate* relevant *complex informational texts* into their instruction. This will require that teachers in each content discipline become proficient in gauging the complexity of informational texts for use with their students. A tool for evaluating text complexity is included as you add informational texts. A tool for *Generating Questions from Considering Qualitative Dimensions of Complexity* is provided to assist teams of teachers in examining literary and informational texts for use in their classrooms. For more information on the use of complex informational texts, refer to: [http://www.corestandards.org/the-standards/english-language-arts-standards/reading-informational-text-6-12/grade-11-12/](http://www.corestandards.org/the-standards/english-language-arts-standards/reading-informational-text-6-12/grade-11-12/). FLDOE provides additional guidance regarding text complexity [here](http://www.corestandards.org/the-standards/english-language-arts-standards/reading-informational-text-6-12/grade-11-12/).

For teams from districts that are incorporating the *Comprehension Instructional Sequence* (CIS), please click here: [http://www.justreadflorida.com/](http://www.justreadflorida.com/)

**Step 3, Task 6: Study Curriculum**

Now that your team has generated a research base relevant to the teaching and learning of the topic or concept that you have selected, it is time to study specific lessons and curricular materials that your district and/or school have adopted for the subject/topic that is the focus of your team’s research. This step includes but is not limited to:

- **Exploring** relevant curricular materials (e.g. textbooks, worksheets, assessments) for exemplary lessons that exist on the topic

- **Identifying** a learning progression that moves students from pre-requisite knowledge and skills to desired standards-based learning outcomes.
Step 4: Plan your Team’s Lesson

As your team has probably come to realize, planning a research lesson is an intense collaborative effort requiring a great deal of research and exploration of both the content and the pedagogy relevant to your chosen research focus. In this step, your team will use all that you have learned through your study of research and your use of formative assessments with your students, to construct a detailed lesson plan that addresses your lesson focus and that fits within a coherent unit of instruction.

Step 4, Task 1: Construct Your Unit Plan
Before writing the lesson plan for your chosen topic, it is important to first consider how this topic fits within a larger unit of instruction. Knowledge of what learning comes before and after the learning outcome intended by your research lesson, is essential to your success in developing a coherent teaching-learning plan.

The “Construct Your Unit Plan” task poses several important questions for your team to consider in the design of your instructional unit. These include items addressing:
- essential prerequisite knowledge and skills
- learning progressions across grades
- learning progressions within grades
- unit learning goals and objectives
- related technology associated with the unit of instruction
- a description of how your team will evaluate student thinking and learning throughout the unit, and
- a narrative or schematic describing the flow/sequence of instruction within the unit.

This resource kit includes a “Developmental Story,” which provides an overview of the learning progression for this topic across grade 5. As a team, study the Developmental Story included below, and discuss how these elements can be included in your team’s unit of instruction on inverse operations.
### Developmental Story for Inverse Operations K-5

<table>
<thead>
<tr>
<th>K-2</th>
<th>G3</th>
<th>G4-5</th>
<th>Algebra 1</th>
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<td>Number line (hopping)-K</td>
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<td></td>
<td>School: numbers w/ratio</td>
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**Step 4, Task 2: Develop Your Research Lesson**

Now that your team has developed a learning progression that maps out the flow of key concepts, skills, and learning objectives that comprise your chosen instructional unit, it is time to select one of the lessons in your unit plan as the research lesson. As you undertake this step, it is important to bear in mind that lesson study affords your team an opportunity to select a research lesson from among the best lessons available. In other words, your team is not being asked to create a lesson from scratch. It is preferable for your team to adapt an existing lesson to the specific demands of your curriculum and the needs of your students.

The Lesson Study Research Group has links to Lesson Study Work Samples ([http://www.tc.columbia.edu/lessonstudy/worksamples.html](http://www.tc.columbia.edu/lessonstudy/worksamples.html)) from a variety of grade levels and subjects. Your team is also encouraged to search the resources available in CPALMS (cpalms.org) for lessons relevant to your team’s research focus.
The Lesson Plan Source on the LSSS’ “Develop Your Research Lesson” page presents your team with the option to:
Regardless of the source of your research lesson, your team should generate a **lesson script** that outlines:

- The **Steps** of the lesson, including any teaching/learning activities in each step
- **Anticipated student responses** to each step, and
- **Points of evaluation** for each step, which may include:
  - Teacher responses to student questions or actions
  - Relevant student data that might provide insight into student learning

A template for generating the lesson script is located in the *Lesson Plan Source* dropdown under the “Create a lesson plan using a basic template within the LSSS” option.

A sample lesson plan is included in this resource kit. Your team may wish to adapt this lesson plan or develop your own, but either way, it is important that you construct your own script, anticipate the responses of your students to the teaching moves in the script, and develop your own evaluation points for collecting student data.

Please examine the lesson plan and the script below:

### 1\textsuperscript{st} Grade Inverse Operations, The Great Cookie Mystery

For the lesson on May 24\textsuperscript{th}, 2013

At Pine Trail Elementary, Mrs. Hajdin’s Class

Instructor: Mrs. Stephanie Hajdin

Lesson plan developed by: Diane LeJeune, PJ Maccio, Lydia Wachtel, Stephanie Hajdin

**Topic: Inverse Operations**

**Brief Description of Lesson:**
In this lesson students will understand the importance of thinking strategically when problem solving. Students are also deepening their understanding of inverse operations.
Title of the Lesson: The Great Cookie Mystery

Goals of Instruction:

Lesson goals for students:
Students will use addition and subtraction within 20 to solve word problems with unknowns in all positions.
Students will begin to understand that when you take away exactly what you put in, you will have what you started with. (Inverse)

Overarching Goals:
• To improve math instruction so that students experience a joy in doing mathematics.
• To improve math instruction so that students experience the confidence and rigor required by the Common Core State Standards.

Product Goals:
• To create a lesson study that will enrich our students’ understanding of mathematics.
• Eventually, to pull together research materials and a series of questions that will help other teacher teams do a mathematics lesson study.

CCSS Math Practice Goals:
• MP#1- Students will make sense of problems and persevere in solving them.
• MP#3- Construct viable arguments and critique the reasoning of others.

Benchmark

MACC.1.OA.1.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Background Information

Pine Trail Elementary services approximately 700 students in grades K-5. This lesson is being taught in Stephanie Hajdin’s first grade class. This class has 18 students of varying abilities. This class consists of ten boys and eight girls ranging in age from 6-8. The math levels in this class vary from students with solid math understanding to progressing and finally there are those students who are emerging in their level of understanding. Currently, there are 4 students who are emerging in their level of understanding.
Developmental Story

Inverse operations - Developmental Story

- What experiences do our students have with this idea of undoing or the idea of opposites beginning with kindergarten and progressing to Algebra 1?
- Is “inverse” meaningful to kids?
- More than one operation needed to use inverse.
- Clarifying the meaning of – (negative), e.g.) -1 X 5 = -5

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Background and Rationale

During collaboration, our team discussed that students in middle school struggle greatly with the concept of inverse operations. There is a need to “plant the seeds” of understanding of the topic of inverse operations with young children as “undoing” something. Our hope is to help students to think strategically when problem solving and deepening their understanding of inverse as it applies to addition and subtraction. In looking for pertinent research regarding this topic with young students, we found a lack of relevant research.

(a) A particular area that we feel students’ need additional practice is in the ability of our students to be able to share their mathematical thinking. In a Kindergarten classroom this can be a struggle for young learners. Our hope is that we can encourage our children to feel safe to explain their thinking.

(b) Mathematical Practice number one: Make sense of problems and persevere in solving them. Mathematical Practice number three: Construct viable arguments and critique the reasoning of others. Students should be able to make sense of the dice game and preserve in finding the different ways to make seven. While playing the game students should be able to justify their answers and recognize when their partners are making mathematical errors.
<table>
<thead>
<tr>
<th>This column shows the major events and flow of the lesson.</th>
<th>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</th>
<th>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</th>
</tr>
</thead>
</table>
| **1. ** **Introduction**  
Teacher: *Okay. These last few days we have been working on how to make the number 5 and the number 6. Let’s review the number 6. Can someone come up and show me how many and how many make 6?*  
Teacher: *Can someone show me a different way?*  
SR: *Yes*  
Student will show one more way.  
Teacher: *Today we are going to talk about the number 7 and the ways to make it.*  
Teacher: *Does that make sense? Is _____’s answer reasonable? If a student says the incorrect answer teacher will use flower counters to prove/disprove any misconceptions. Teacher will have child come up to prove/discuss correct answer also.*  
Teacher: *How do we know if students understand the task and are motivated to solve it? What prior knowledge is sparked? Students accurately create ways to make 6.* | **Teacher: ** *Does that make sense? Is _____’s answer reasonable?*  
If a student says the incorrect answer teacher will use flower counters to prove/disprove any misconceptions. Teacher will have child come up to prove/discuss correct answer also.  
**Teacher will have child come up to prove/discuss correct answer also.* | **How do we know if students understand the task and are motivated to solve it? What prior knowledge is sparked? Students accurately create ways to make 6.** |
| **2. ** **Activity**  
This section describes the activity and the anticipated student responses (including incorrect solutions and places where students might get stuck).  
Teacher: *I want you to look at this picture .(on top of student textbook p. 51) What do you see?*  
SR: *I see numbers on a cube. I see a 6. I see a 4. I see a 5. I see a 1. I see 2 kids with blocks with numbers.*  
Teacher: *What do you think they are doing?*  
SR: *Playing a game.*  
Teacher shows words with little girl saying “It is 4”.  
Teacher: *Now what do you think is going on in the picture?*  
Teacher shows next picture and reads talking bubbles.  
Teacher: *She rolled a 4. Which number does the boy need show to make 7?*  
SR: *3.*  
Teacher: *Well today we are going to play the game like the children.*  
Teacher models rolling dice. Talks about what she rolled.  
Teacher will use the blue flower counters to model  
**How will the teacher will handle the different student responses, especially incorrect solutions, students who get stuck, or students who finish early? What are the specific questions or comments the teacher will pose in each case?**  
If a student says the incorrect answer teacher will use flower counters to prove/disprove any misconceptions. Teacher will have child come up to prove/discuss correct answer also.  
**Teacher will have child come up to prove/discuss correct answer also.** | **If a student says the incorrect answer teacher will use flower counters to prove/disprove any misconceptions. Teacher will have child come up to prove/discuss correct answer also.** |
| **Teacher: ** *How will the teacher will handle the different student responses, especially incorrect solutions, students who get stuck, or students who finish early? What are the specific questions or comments the teacher will pose in each case?* | **Teacher will have child come up to prove/discuss correct answer also.** | **Teacher will have child come up to prove/discuss correct answer also.** |
### Solution Strategy

**Teacher:** How many more counters do I need to make 7 if I have____?

**SR:** The correct answer, the teacher’s roll or any other number to 7.

**Teacher:** Prove it.
Teacher will model one more time.
Teacher would discuss rules of taking turns, play and record on the board.

**Teacher:** You will only roll one of the die. The person who rolls will write the number on the first box. You and your partner will figure out how many more you need to make 7. Your partner will use the second die to show how many more you need to make 7. You will have 7 counters at your table if you need them.

**Teacher:** We need to have a way to record all the ways that you find. So you will use this recording sheet to show all the ways.

**Teacher:** Okay, at your seat you have the dice and a recording sheet that is just like the ones we used up here. The person who rolls the dice is the person who will write on the worksheet. You will take turns rolling the die and recording.

Teacher will send students in groups of two back to their seats to complete activity and record all responses.

Teacher will monitor individual groups. Teacher will observe student responses and choose teams to present their findings of ways to make 7.

### 3. Compare and Discuss

Teacher will call the children back to the carpet with their recording sheets.
While observing students teacher will mark each groups answer for sharing with a * on their skill sheet. (This will organize the presentation of solutions)
Teacher will choose students to explain/present their thinking in the following way: 1 and 6, 2, and 5, 3 and 4, 4 and 3, 5 and 2, 6 and 1.

**Teacher:** How did you know you needed a ___ when you rolled a ______?

**SR:** I counted

**Teacher:** Can you show me how many and how many make 7?

Students may try to roll second die. Teacher will explain that the second die is to be placed down instead of rolled.

Students who finish early will continue to play but not record results.

Students may use IMM (immediate recall), ON-F counting on from 1, 2, 3, 4, 5, 6, using fingers, CT (using counters/dot cards)
Students may record 7 on the recording sheets instead of the parts that make up 7.

T: Did you roll a 7? Can you me show me how you got the 7? Do you want to change it?

**Teacher will have 7 dot cards available to students who need extra support.**

Ideas to focus on during the discussion are: If one way to make 7 is not given the teacher will continue lesson and see if someone notices the pattern at the end of the lesson.

If a way to make 7 is missed T may ask: Does anybody notice anything that might be missing

Indications that students are benefitting from the discussion is their ability of the children to decompose and compose numbers to 7. They can manipulate semi-concrete objects and link it to the
SR: I just knew
SR: I used the counters and put them on the dot card. Then I found out how many more I needed.
Teacher will put strips with combinations next to the numerals to link the abstract with the concrete.

[4. Apply the learning to another activity]
- Teacher will play game with teacher showing a numeral card and students telling the complement of seven. Students will then complete the bottom of the textbook p 51
- Teacher could use a situational story problem for students to discover the ways to make 7 to check for student understanding.

Questions that will help students understand include:
Teacher will ask students if this activity reminds them of something else that they’ve already done in class.

Students will be able to decompose and compose other numbers greater than 7 using their prior knowledge.

[5. Consolidate the learning]
- Students will explain how they got the complement of the number needed to make 7.

T: Who can tell me what we learned today?
T: Wow! I am so impressed with all the ways you were able to make 7.

Prompts for discussion may include:
Student explanations of different ways to compose/decompose 7.

Explanations and comments that indicate the correct way to decompose/recompose 7. Explanations using the dots cards to show thinking.

Board Plan

I rolled a __ I need a __
to make 7.

This chart will have 6 sets of dice that will line up with the dots.

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Reflection Questions

- Can students easily and readily decompose 7 into two parts?
- Do students have an automatic recall of ways to make 7?
- Can students readily share mathematical thinking?
- When students have difficulty, do they persevere in trying to solve the problem?

Summary of post-lesson reflection

This lesson on the structure of seven was the first introduction into the ways to make the number seven. In a previous lesson students have counted to seven and have seen the parts of seven while looking at photographs, but have not formally been introduced into the multiple ways to make the number seven. In post lesson discussion of this lesson the team noticed many things. The beginning of the lesson started out well with the students being able to come to the board to show some of the ways to make the number six which had been taught in previous lessons. Lydia Wachtel modeled the expectations of rolling the die and then “placing” the second die to show the number needed to get to seven. Once students got to their seats to play the game it became evident that using the dice was causing several problems for the students. Students were able to roll the first die and record it on their papers. However, many students tried to also roll the second die and simply record that number as the complement to seven. They did not seem to understand that the second number rolled didn’t necessarily get you to the number seven. To our team, it seemed that the manipulative was getting in the way of the students’ learning. As extra support students were given a ten frame with seven on it to assist struggling learners. Every group except one needed that extra support. Once Mrs. Wachtel gave the ten frame card to the children they all seemed to be able to find the ways to make seven.

After playing the game for several minutes Mrs. Wachtel called the children up to the board to share their findings. During this share out Mrs. Wachtel purposefully called the children up to show one of their ways. (See board diagram on previous page) During this part of the lesson there was some rich discussion into the patterns that the children noticed on the recording sheet.

After observing Lydia Wachtel teach our lesson on composing and decomposing 7, our other kindergarten teacher Lisa Gilbert, taught the lesson to her students. After discussion from the group, the following alterations were made. The second die was eliminated because this seemed to cause great confusion with the students and got in the way of overall understanding of composing and decomposing 7. Two sided counters were given (red and yellow). Students were instructed to roll one die. The partner was then asked to flip over the counters to the number that was rolled. For example, if a five was rolled, the partner would flip over 5 counters to yellow and 2 would still be red. They would tell their partner that 5 and 2 made seven. Then the students would switch roles until each had several turns rolling and counting.

Discussion of reflection questions:
• Can students easily and readily decompose 7 into two parts?
At this time overall the students cannot readily decompose seven into two parts. Children will need additional practice to reach this point.

• Do students have an automatic recall of ways to make 7?
Not at this time. We believe that the students will need several opportunities to work with seven. The manipulative used in this lesson (dice) was hindering the students’ abilities to focus on the ways to make seven.

• Can students readily share mathematical thinking?
Although the camera seemed to cause a few of the students to become nervous, overall we were pleased with the children’s abilities to share their mathematical thinking. During the sharing of the ways to make seven the children were able to discuss the ways they made seven and were also able to prove it to the class.

• When students have difficulty, do they persevere in trying to solve the problem?
This group of children does appear to persevere when trying to solve problems. Children attempted to solve the problem given to them. There wasn’t one group who gave up on the task. However, we do believe that some children didn’t know that they were not making seven using two parts. Therefore, they didn’t have the necessary skills needed to be successful at this task. More time was spent on the game, not necessarily the meaning of the number seven.
Summary of learning from lesson study cycle

Please post or distribute Item 12 so that each team member can write an individual response, and please include the individual responses when you return the video card and the artifacts from your lesson study cycle. After team members have written individually, share ideas and summarize here your learning as a group.

Your ID number:___________
What we learned about composition/decomposition of numbers to 10 and our teaching of it:

What we learned about students’ development of mathematical practices:

Implications for our next lesson study cycle (for example, what makes a good topic to investigate, modifications to the lesson study process, etc.)
<table>
<thead>
<tr>
<th>Lesson Step</th>
<th>Anticipated Student Responses</th>
<th>Evaluation Points/Teacher Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think about… Discuss the different ways students took to recess.</td>
<td>What are short cuts in their lives?</td>
<td>Students understand that route #2 is the quickest way to recess.</td>
</tr>
<tr>
<td>T: What are the 3 different ways we took to recess?</td>
<td>S: Although all ways will get you to recess. The 2nd route is the shortest/fastest way.</td>
<td></td>
</tr>
<tr>
<td>T: Which was the fastest way to recess? Be prepared to tell why.</td>
<td>T: So the second way gets us to where we want to go quicker. (Students should understand that one of the routes gets them to recess faster than the rest)</td>
<td></td>
</tr>
<tr>
<td>T: What are short cuts in their lives?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S: Although all ways will get you to recess. The 2nd route is the shortest/fastest way.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T: I have a little mystery that I need your help to solve.</td>
<td>S: Mr. H put 5 cookies to start in your lunchbox. You woke up and put 17 more (the rest of the cookies) in your lunchbox.</td>
<td>If a student thinks there are 5 or 17 total cookies to start teacher will need to clarify with: So how many cookies did Mr. Hajdin put in my lunchbox? Were 5 cookies all that I had in my lunchbox or were there…</td>
</tr>
<tr>
<td>T: Last night I made the yummiest chocolate chip cookies. Mr Hajdin put 5 of the cookies in my lunchbox because he knows how much I love them. But, around midnight, I woke up and put the rest of the cookies in my lunchbox… 17 of them. When I opened my lunchbox this morning, I found a note: Dear Sweetheart, There were no cookies in the cookie jar this morning. I checked your lunchbox and there were all the cookies! I ate a few of them. See if you can figure out how many cookies I ate.</td>
<td>T: Tell me what is happening in our story?</td>
<td></td>
</tr>
<tr>
<td>S: Mr. H put 5 cookies to start in your lunchbox. You woke up and put 17 more (the rest of the cookies) in your lunchbox.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**CLARIFYING QUESTIONS:**

- S: Mr. H put 5 cookies to start in your lunchbox. You woke up and put 17 more (the rest of the cookies) in your lunchbox. If a student thinks there are 5 or 17 total cookies to start teacher will need to clarify with: So how many cookies did Mr. Hajdin put in my lunchbox? Were 5 cookies all that I had in my lunchbox or were there…
T: So how many cookies did Mr. H give me?
T: Were the 5 cookies all that I had in my lunchbox?
T: Where did the rest of the cookies come from?

T: Let’s see if we can write this as a math sentence with numbers and symbols. (Write the 5 on the board.)
T: Mr. Hajdin put in 5 cookies. (Write the + 17 on the board.)
T: Next, I put in 17 cookies. (Write the 5+17 on the board.)
T: What happened next? Do we know how many he ate?
T: Something is missing. How should we record that Mr. Hajdin ate some cookies, and we don’t know how many? (Put the - ? in the math sentence.) 5+17 - ?
T: What other information do we need to know?
T: Right. Let’s count how many are still in the lunchbox.

Open the prepared lunchbox.
The teacher then counts the cookies left in the lunchbox.

T: So now, I have 17 cookies.

S: Mr. Hajdin ate some. No.
   Count how many we have left.
S: Use a – symbol because he ate some.
   Put a ___? because we don’t know how many he ate.
S: = sign, 5+17=

S: We could put a box to show that we don’t know how many cookies Mr. H ate.
S: We need to know how many cookies are in the lunchbox.

S: equal sign (=) and put 17 after it.
S: 22

Teacher clarifies 5, 17, +
T: What happens when I take something away? Does it get bigger or smaller?

T: Where does it show that Mr. Hajdin ate some?

Clarify misconceptions with guiding questions:

T: Show me where the 22 cookies are?
How can we finish our math sentence to show that at the end of our story we have 17 cookies?

**Teacher will write:**
5 + 17 - ? = 17.

**POST ON BOARD AND READ:**

**What number would replace the ?**

**T:** *Think for one minute about how to solve the problem.*

Give 1 paper for each pair of students to take back to their seats.

(***Ask 1 partner to write the equation and the other checks to make sure it’s written correctly.**)

**T:** *Put your pencils down.*

**T:** *These are the math tools we have in our classroom you may choose to use.*

Possible tools that teacher may have available for students are 100 boards, 10 frames, counters, number lines, or students may choose to use a mathematical drawing.

**T:** *Remember a math tool*

Where are the 22 cookies? We just counted 17 cookies. How can there be 22 cookies? Teacher and students can recount cookies if necessary.
can also be a math drawing or picture.

**T:** Discuss with your partner what strategy you are going to use to solve the problem.

**T:** After you decide on a strategy, you can get started on solving the problem.

**T:** Please show your thinking. Be ready to explain your strategy.

Give the students about 5 minutes to solve the problem.

Gather the students to carpet to share strategies.

Possible student solution strategies for sharing: (from most concrete to abstract)

1. **Show a strategy that uses manipulatives.**

   **T:** Why does this work? Can you explain why it works?

2. **Show a strategy that uses a drawing.**

   Drawing strategy would be the same as above, but with a picture.

Students could use counters. They will pull out 5 and 17. Some might put all counters together and then separate 17 out leaving the 5.

Some might draw the 5 and the 17. Some might cross out the 5.

Some might draw 5 and 17 as one group. Then count out 17 and take

Some students may use a ten frame.

Some students may use a number line.

If someone immediately says 5, the teacher asks them to prove it.

I started out with 5 counters, then I took 17 more because Mrs. H put them in her lunch box. So now I have 22 counters. Then I took counters away until I had 17 left. I knew that I took 5 away, so that must be the amount of cookies that Mr. H ate.

Remember, a strategy is efficient if it works all the time.
<table>
<thead>
<tr>
<th>T: Why does this work? Can you explain why it works?</th>
<th>I looked at the number sentence and saw that we started with 5 and added 17. Then we needed to end up with 17, so I just took the 5 away that we started with.</th>
<th>If students don’t automatically think this is the most efficient way, teacher should stress: Students that quickly recognize the pattern will be able to find the missing part. Students may not look at the equation carefully and see that the first addend is what we are left with.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3. Show a strategy that just the equation.</strong> T: Why does this work? Can you explain why it works?</td>
<td>Teacher will have magnetic cards with the numbers /symbols 5, 17, +,=, ? and 17 written so that teacher can emphasize the point that the equation solution is the quickest/most efficient way. (Teacher will physically pull the 5 card and the ? to show that what is left is 17 on each side of the equation)</td>
<td></td>
</tr>
<tr>
<td>T: Which strategy is the most efficient or quickest way to solve the problem?</td>
<td>The equation is the fastest and easiest way.</td>
<td></td>
</tr>
<tr>
<td>T: Wow, You are such smart mathematicians! Here is a new challenge for you! 7 +19 - ? = 19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T: Take your paper back to your seats. Use the back of your paper to solve this new challenge.</td>
<td>Allow students time to</td>
<td></td>
</tr>
</tbody>
</table>
solve problem.
Bring students to carpet to share solutions for above equation.

T: Amazing

T: Let’s see if you can solve this one right here on the carpet.
11 +8 - ? = 11

Students share solutions.

T: Just how did you find that solution so quickly?

(If students are ready, use next equation.)
(If not, conclude lesson)
T: Okay, here is one really tricky challenge.

Try this:
4+3+17-?=17

T: So you are taking away exactly what you put in and you have what you started with. Wow! That is really important! I am so impressed that you are thinking like a mathematician!

Teacher will restate the importance of the most efficient/quickest strategy.
Post: When you take away exactly what you put in, you will have what you started with.
The Great Cookie Mystery!
Write the equation from the board in the box below. Solve and be prepared to tell what you did.
Student Work Samples

Examine the student work sample below and consider the following questions:

What strategies did the student seem to use in solving this problem?

What types of strategies do you expect your students to use?
The Great Cookie Mystery!

Write the equation from the board in the box below.
Solve and be prepared to tell what you did.

\[ 5 + 17 - 5 = 17 \]

[Diagram of cookies]
Step 5: Teach your Team’s Lesson

The weeks that your team has devoted to the research and preparation of your teaching-learning plan for this lesson study cycle are nearing an end. Prior to teaching and observing the lesson, there are several important things for your team to decide.

- When and in what class (or classes) will the lesson be scheduled?
- Who will teach and/or re-teach the lesson?
- What materials, equipment, and supplies are needed for the lesson?
- How will substitute teachers be provided to cover your team’s classes?
- What school/district/other personnel will be invited to observe the lesson?
- When and where will the debriefing and re-teaching phases be held?

Step 5, Task 1: Devise a Data Collection Plan

Catherine Lewis (2002) has referred to the data collection plan as an element of lesson study that “cuts across all three circles” (p.66) of the teaching-learning plan. The data that your team chooses to focus on during the observation of the lesson should provide insights into
student learning of the lesson goals, goals of the larger unit of instruction, and the larger over-
arching goals stated in your research theme.

The Data Collection Plan page in Step 5, Task 1 of the LSSS prompts your team to
consider the following in devising their data collection plan:
- The immediate goals of the research lesson
- The broader learning goals of the instructional unit
- The over-arching student goals expressed in your team’s research theme
- Relevant samples from student work on the research lesson,
- Individual data collection assignments for teacher/observers during the
teaching of the research lesson

Step 5, Task 2: Assemble Lesson Materials and Teach the Research Lesson
Your team must decide upon what classroom(s) the teach and/or reteach will take place, what
materials will need to be prepared in advance for the students, and who will be attending the
teach as data collectors and observers. Once these issues have been resolved, your team will
need to ensure that everything is in place on the day of the lesson.

So that there is no miscommunication among team members, it is wise for the team to
allocate responsibilities for obtaining instructional materials and setting up the classroom
space(s) for the teaching of the research lesson, to specific team members.

A Materials Inventory Tool is included in the Instructions panel in this step of the LSSS to
document all materials and equipment needed for the lesson as well as the team member(s)
responsible for obtaining them.

Teach your Team’s Lesson

As your team prepares to teach its research lesson, it is important to attend to the tasks listed
below.

Before the Teach:

- All observers including visitors should be notified of the teach-debrief-reteach schedule
  and given directions for the location of the teaching session and post-teach meetings.
- A data collection strategy should be agreed upon by the team and all observers should
  be briefed on their data collection responsibilities.
- All instructional materials should be in place for the teaching session and the classroom
  should be prepared accordingly.
- Copies of the lesson script should be made available for each observer, including
  visitors.
- All school administrators should be notified of the schedule and invited to attend.

During the Teach:
• Observers should position themselves about the classroom in accordance with the team’s data collection strategy. Depending upon the team’s preferences, observers may choose to focus on one group of students or may move from group to group.
• Observers should NOT talk to or interact with students.
• Observers should focus upon the actions of the students and how they receive the lesson.

After the Teach

• All team members and observers should refrain from discussing their data and observations until the debrief meeting is convened.

It is suggested that the team break for at least 15 minutes before convening for the debrief. Each person may organize his or her data individually during this time. It is recommended that the team members refrain from discussing the lesson and any data or observations until the debrief.

Step 6: Debrief and Reflect

The lesson debrief provides your team with an opportunity to discuss data collected during your observations of the lesson and implications for student learning. The debriefing should focus on two main themes:

• Discussion of student data and reflection by each team member and observer
• Discussion of how the lesson can be modified to better address desired learning outcomes

The Facilitator should begin the debriefing by thanking the teacher who taught the lesson on behalf of the team and briefly describing the Debriefing Protocol located in the instructions pane on Step 6, Task 1 of the LSSS.

After the first round of discussion, in which each team member and observer shares their data and reflections without interruption, the Facilitator should move the debrief to the general discussion phase, in which the team and invited guests begin discussing ways that the lesson can be improved to better achieve the learning outcomes envisioned in the teaching-learning plan.

Step 6, Task 1: Debrief Teaching Session

After team members and observers have shared their data and reflections without interruption, the Facilitator should move the debrief to the general discussion phase, in which the team and invited guests begin discussing ways that the lesson can be improved to better achieve the learning outcomes envisioned in the teaching-learning plan.
A Lesson Plan Modification Tool is provided for team members to document proposed changes and to generate a revised Research Lesson Plan for re-teaching.

Re-teach the Research Lesson (optional). The modified research lesson may be taught to a separate group of students, preferably by a different teacher. You can use the same data collection plan that your team used when the lesson was originally taught. You may want to revise the plan, though, if you feel that it did not allow for effective data collection or to bring it into better alignment with the modified lesson. It is optimal if the re-teach is conducted on the same day as the original research lesson.

Step 6, Task 2: Debrief Teaching Session

A final debrief and reflection session should be scheduled after the reteach. After the initial discussion of the data collected during the re-teach, the general phase of the discussion should focus on reflecting upon what was learned by your team over the course of the cycle and how this knowledge can be useful in planning and conducting subsequent cycles. Finally, your team should discuss your plan for reporting your activities over the course of the cycle. Your team can report what it has learned over the course of the LS cycle and how that knowledge can be useful in further instruction, in the dialogue box in Task 2 of Step 6 of the LSSS.

Step 7: Report and Share your Team’s Lesson Study Work

Your team has the option of sharing its work, including any final documents that it has produced over the course of the lesson study cycle.

In addition, CPALMS’ LSSS provides reporting options that include:

- Storing your lesson study cycle documents and generating a report for your team, and/or school and district officials who require one for in-service credit, and
- Submitting any, or all, of your unit plan and research lesson to CPALMS (cpalms.org) for consideration as a CPALMS instructional resource or lesson study resource kit.
References

